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Short Note

Formula and nomogram for estimating the number of regularly patterned elements on the surface of a spheroidal microfossil

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Many spherical microfossils have regularly patterned surface features, the number of which can be useful for precise identification (Faegri and Iversen, 1989). These features may be pores, spines, verrucae, or other sculptural or structural elements. For example, the number of pores on a grain of chenopod pollen may be used to narrow the list of possible species (Faegri and Iversen, 1989). McAndrews and Swanson (1967) proposed a geometrical model relating the number of pores on a periporate pollen grain to the ratio of the distance between the centres of pores (chord) and the diameter of the pollen grain. They also suggest that since the chord/diameter (C/D) ratio is a function of the feature density, this ratio may be used in place of feature number in identification keys. However, with the exception of their own paper, this plan has not been carried out in other keys. Their model, which can be applied to any spherical object with regularly patterned surface features, produced a set of equations, which culminates in the approximate number of pores.

Christensen (1986) corrected McAndrews and Swanson's published equations, which contained a typographical error, and further simplified them. The basis of Christensen's method is nevertheless the same as McAndrews and Swanson's method, i.e. estimating the number of triangles with a specified length of side which can be placed on the surface of a sphere, and the results are very similar.

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The corrected and simplified formulas are:

$$a = 2\arcsin(C/D) \quad (1)$$

$$A = \arccos[(1-\cos(a))/\sin(a)\tan(a)] \quad (2)$$

$$N = 2/(1-\pi/3A) \quad (3)$$

where a is the side of an equilateral triangle on the surface of the sphere with three neighbouring pores as vertices, in radians; C is the chord distance between pore centres; D is the diameter of the pollen grain; A is the internal angle of the triangle; and N is the number of pores which can be placed on the sphere.

While this set of equations gives a maximum number for the population of pores, it is somewhat unwieldy, and in practice produces large overestimates of the pore population (Table I). For convenience, McAndrews and Swanson (1967) and Christensen (1986) modelled the patterned sphere as a polyhedron with equilateral triangle faces, which they recognized as a geometrical impossibility. It is this simplifying assumption which causes the estimated numbers to be too high. Another approach was tried by Hanks and Fryxell (1979), which although having the virtue of great simplicity, produced severely inaccurate estimates for most cases. Their formula is:

$$N = (D/C)^2, \quad (4)$$

which in essence reduces the sphere to a two-dimensional surface.

In this note, a geometrical model of intermediate

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TABLE I

Actual and estimated numbers of pores on periporate pollen grains

D	C	C/D	Cn	$H+F$	$M+S$	*This paper*
6.1	1.1	0.180	55	31	110	51
3.1	0.6	0.194	52	27	96	48
6.8	1.9	0.279	34	13	45	33
4.1	1.5	0.366	28	7	26	25

D = grain diameter, C = chord distance, Cn = number of pores estimated by counting one hemisphere and multiplying by 2, $H+F$ = number of pores estimated from formula (4) by Hanks and Fryxell (1979), $M+S$ = number of pores estimated from look-up table in McAndrews and Swanson (1967), *This paper* = number of pores estimated from formula (6). Units for D and C are arbitrary.

complexity, with a nomogram, produces estimates of a regularly patterned surface feature number which match the real number within the limits of measurement error. A spheroidal microfossil can be modelled as a sphere enclosing rays with equal angles between them. A sphere has 1080 internal degrees (or 6π radians). If the repeating elements on the surface are arranged so that the rays passing through each element make the same angle 2θ with each nearest neighbour, then the number of such rays and hence the number of the repeating elements on the surface of the sphere is constrained by the angle 2θ between the rays and the sum of 1080 degrees in a sphere. Such an arrangement will occur wherever the chord distance between features is constant. Where the arrangement is sub-regular, the chord distances and the angles will be sub-equal. In such a case, the average chord distance will yield the average angle, which will suffice for an approximation of the number of regularly patterned features. By determining 2θ and dividing it into 1080, it is possible to estimate the number of regularly patterned features on the surface of the sphere.

The angle θ can be estimated from the chord distance (C) and the grain diameter (D), using the trigonometric relationship:

$$\theta = \arcsin(C/D) \qquad (5)$$

Multiplying by 2 to obtain 2θ and dividing into 1080 yields:

$$N = 540/[\arcsin(C/D)] \qquad (6)$$

which gives the number of regularly patterned surface elements.

This equation is simple enough to be used with a desk calculator beside the microscope. A nomogram for estimating the equation for values of C and D ranging from 1 to 100 is shown in Fig. 1, and an example in Fig. 2. Clearly, the higher the number of regularly repeating features, the more critical the precision of the measurements. Table I compares the results of tests using this formula and that of McAndrews and Swanson (1967) with actual pollen grains, where the number of pores on one hemisphere was counted and multiplied by 2 (Monoszon, 1952, in McAndrews and Swanson, 1967). The error in this model is very small, and well within the limits of measurement error, which from standard deviations in the measurements of actual grains in McAndrews and Swanson, is likely to be on the order of 10%. This measurement

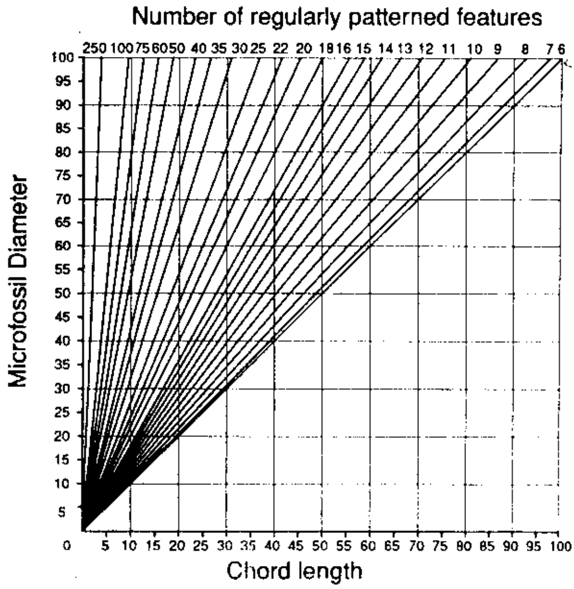

Fig. 1. Nomogram for estimating regularly patterned feature number. The units for D and C may be microns, μm divisions, or any other convenient unit. If the use of a particular unit causes the point (C, D) to plot outside the nomogram or in the crowded bottom left corner, the measurements may be multiplied or divided by a convenient factor to move the point into the main body of the nomogram (for example, all C and D values in Table I could be multiplied by 10).

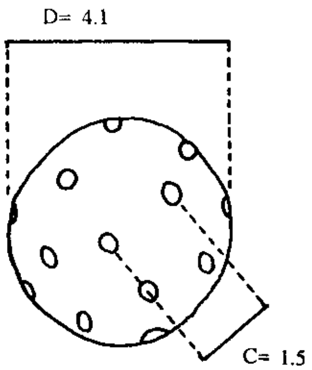

Fig. 2. Both *C* and *D* should be measured several times, and the average values used. It is important that *C* be measured in the plane of the slide, so it may be best measured near the middle of the grain or at the periphery; measurement elsewhere will be affected by perspective.

error can of course be substantially reduced by using the averages of several measurements. Since the measurements used here are the same as those used by McAndrews and Swanson (1967), we can use their conclusion that measurement error has a theoretical potential of 25%, but rarely reaches so high due to cancelling of opposing errors. It can be seen from the nomogram (Fig. 1) that small errors in measurement of the chord length can result in large errors of feature number estimate, but that the measurement of the sphere diameter is less critical, particularly at high feature numbers.

Although the model developed here is for spheroidal microfossils, it may also be applied to ellipsoidal objects by using the mean of the two diameters of the ellipsoid. This will entail a small increase in the error of the estimates. On the other hand, features which are not flush with the surface of the sphere must be measured with caution. The model depends on the chord distance being measured accurately where it intersects the sphere surface; if the features are indented or projected, this may be difficult to determine. One solution is to simply estimate the point of intersection of each feature with the sphere circumference.

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